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Dominating Sets Of Divisor Cayley Graphs

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Abstract—Let n ≥ 1 be an integer and S be the set of divisors of n .Then the set S*= {s, n- s / s $\in$ S, n ≠ s} is a symmetric subset of the group (Zn, $\oplus$), the additive abelian group of integers modulo n. The Cayley graph of (Zn, $\oplus$), associated with the above symmetric subset S* is called the Divisor Cayley graph and it is denoted by G(Zn , D) .That is, G(Zn , D) is the graph whose vertex set is V={0,1,2,…,n-1} and the edge set is E = {(x , y) / x-y or y-x is in S*}. Let G (V,E) be a graph. A subset D of V is said to be a dominating set of G if every vertex in V \ D is adjacent to a vertex in D. A dominating set with minimum cardinality is called a minimum dominating set and its cardinality is called the domination number of G and is denoted by ϒ (G) .
Index Terms— Divisor Cayley Graph, dominating set, domination number.

1. INTRODUCTION

Berge[4] and Ore[12] are the first to introduce the concept of domination and they have contributed significantly to the theory of domination in graphs. E.J. Cockayne and S.T. Hedetniemi [6] published the first paper entitled "optimal domination in graphs". They were the first to use the notation $\gamma(G)$ for the domination number of a graph, which subsequently became the accepted notation. Allan and Laskar [1], Cockayne and Hedetniemi [6], Arumugam [3], Sampath Kumar [13] and others have contributed significantly to the theory of dominating sets and domination numbers. An introduction and an extensive overview on domination in graphs and related topics are given by Haynes et al. [8]. In the sequel edited by Haynes, Hedetniemi and Slater [9], several authors presented a survey of articles in the wide field of domination in graphs. Cockayne, C.J et.al [5] introduced the concept of total domination in graphs and studied extensively. The applications of both domination and total domination are widely used in Networks.

In this chapter we discuss dominating sets of Divisor Cayley Graphs. Here we have obtained these sets for various values of n . In certain cases we could get minimal dominating sets. In other cases, we are unable to give the proofs to find the minimal sets by the technique we adopted to find these sets. This is because, the divisors depend on the number n and unless we give the value for n, we are not known the divisors and hence the elements in S*.

Hence we have devised an algorithm which finds all minimal dominating and total dominating sets for all n. Our algorithm finds all closed and open neighborhood these sets which cover all the vertices of G(Zn, D). Each theorem is strengthened with examples and the Algorithm is also illustrated for different values of n.

2 DOMINATING SETS OF DIVISOR CAYLEY GRAPHS

Let us define a dominating set as follows

Definition : Let G (V,E) be a graph. A subset D of V is said to be a dominating set of G if every vertex in V \ D is adjacent to a vertex in D.

A dominating set with minimum cardinality is called a minimum dominating set and its cardinality is called the domination number of G and is denoted by γ(G).

We now find dominating sets of Divisor Cayley graph.

Theorem 2.1: If n is a prime, then D = {$rd_0/0 \leq r \leq k-1$, where k is largest positive integer such that $rd_o < n$ and $d_o = 3$}, is a dominating set of G (Z_n, D).

Proof: Let n be a prime. Then G (Z_n, D), is an outer Hamilton cycle.

Let D = {$rd_0/0 \leq r \leq k-1$, where k is the largest positive integer such that $rd_o < n$, $d_o = 3$}. If we construct D, then the vertices of D are in the form {0,3,6,9,….}.

Since G (Z_n, D) is an outer Hamilton cycle, each vertex in D is adjacent with its preceding and succeeding vertices. This implies that every vertex in V-D is adjacent with at least one vertex in D.

Therefore, D becomes a dominating set of G (Z_n, D).

Also by the construction of D, it is clear that D is minimal since no proper subset of D is a dominating set and hence |D| = k is the domination number of G (Z_n, D).

Example 2.2 : Consider G (Z_7, D) for n = 7 which is a prime.

Then S ={1,7} and S* = {1,6}.

The graph G (Z_7, D) is 2 – regular.

The graph of G (Z_7, D) is as follows.

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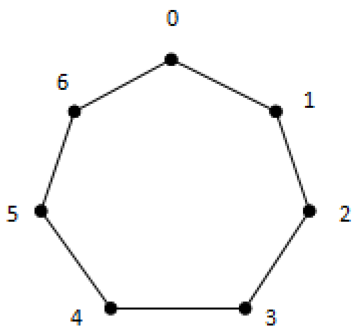

Fig. 1 : G (Z_7, D)

Also, G (Z_7, D) is an outer Hamilton cycle.
If we construct D, we get D = {0,3,6}. We observe that the set D = {0,3,6} dominates all the vertices in V–D. Further, this D is a minimal dominating set because if we delete one vertex from this set then the remaining vertices cannot dominate the vertices of G(Z_7, D) .
Therefore, the domination number of G (Z_7, D) is 3.
A possible number of MDSs of G (Z_7, U_7) is 7 and these sets are given by
{0,3,6}, {1,4,0}, {2,5,1}, {3,6,2}, {4,0,3}, {5,1,4}, {6,2,5}.
Remark : The domination number of G(Z_n, D) is 1 if n = 2,3,4 and 6. The symmetric subset S^* contains all the vertices of G(Z_n, D) for all value of n and hence the difference of any two vertices is in S^*. Thus every vertex is adjacent to all the vertices of G(Z_n, D) so that it is complete. So, every single time vertex is dominating set and hence the domination number is 1.
Theorem 2.3 : Let $n = p^2$, $p \neq 2$ be a prime. Then D = {r d_0 / $0 \leq r \leq k-1$ where k is the largest positive integer such that $rd_0 < n$ and $d_0 = 3$} is a dominating set of G(Z_n, D).
Proof : Consider the graph G(Z_n, D).
Let $n = p^2$, $p \neq 2$ be a prime.
In this case, S = {1, p, p^2} and S* = {1, p, n-1, n-p}.
i.e., S* = {1, p, p^2-1, p^2-p}.
The graph G(Z_n, D) is │S*│ - regular. That is the graph is 4 - regular.
Let D = {rd_0 / $0 \leq r \leq k-1$ where k is the largest integer such that $rd_0 < n$ and $d_0 = 3$}.
We now show that D is a dominating set of G(Z_n, D).
Clearly, the vertex 0 is adjacent to the vertices 1, p, p^2-1, p^2-p as their difference is respectively 1, p, p^2-1, p^2-1 which belongs to S^*.
The vertex 3 is adjacent to the vertices 4, p+3, 2, p^2- p+3.
The vertex 6 is adjacent to the vertices 7, p + 6, 5, p^2- p+6.
This is expressed in the following.

$0 \rightarrow 1, p, p^2-1, p^2-p$
$3 \rightarrow 4, p+3, 2, p^2-p+3$
$6 \rightarrow 7, p+6, 5, p^2-p+6$
⋮

Here we observe that the vertices {0, 3, 6, 9, 12,……. } are dominating the vertices 0, 1, 2, 3, 4, 5, 6, 7 …. .
If we give the value for p then the rest of the vertices which are dominated by {0, 3, 6, 9,……} will also be found.
Likewise the vertices in D dominate the vertices of the graph G(Z_n, D) and hence D is a dominating set of G(Z_n, D).
mple 2.4: Let n = 9.
Then S = {1,3,9} and S^* = {1,3,6,9}.
The graph of G (Z_9, D) is as follows.

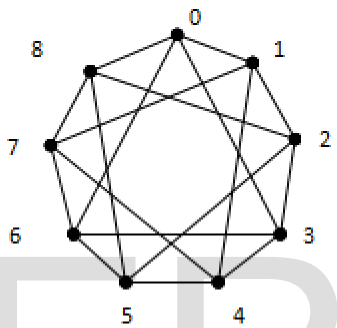

Fig. 2 : G (Z_9, D)

Then D becomes {0, 3, 6} and the vertices adjacent to D are
$0 \rightarrow 1, 3, 8, 6$
$3 \rightarrow 4, 6, 2, 0$
$6 \rightarrow 7, 0, 5, 3$
i.e., the set {0, 3, 6} dominates the vertices {0, 1, 2, 3, 4, 5, 6, 7, 8, 9}.
Hence D = {0, 3, 6} becomes a dominating set of G(Z_9, D).
This set is also minimal.
If we delete any one vertex from D, then the resulting set will not be a dominating set of G(Z_9, D) since the graph is 4 – regular, a set of 2 vertices cannot dominate all the vertices of the graph.
Example 2.5: Suppose n = 25.
Then S = {1,5,25} and S^* = {1,5,20,24}.

The graph of G (Z_{25}, D) is as follows.

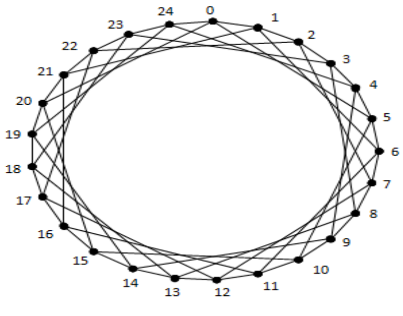

Fig. 3 : G (Z_{25}, D)

Now D = {0, 3, 6, 9, 12, 15, 18, 21}.
The vertices adjacent with the vertices of D are
$0 \rightarrow 1, 5, 24, 20$
$3 \rightarrow 4, 8, 2, 23$
$6 \rightarrow 7, 11, 5, 1$
$9 \rightarrow 10, 14, 8, 4$
$12 \rightarrow 13, 17, 11, 7$
$15 \rightarrow 16, 20, 14, 10$
$18 \rightarrow 19, 23, 17, 13$
$21 \rightarrow 22, 1, 20, 16$
Here we observe that the set D = {0, 3, 6, 9, 12, 15, 18, 21} dominates all the vertices of the graph G(Z_{25}, D).
Since the graph is 4-regular we should have $|D| \geq 7$.
But here we have obtained $|D| = 8$.
That is D is not minimal.
To get minimal dominating sets, we give the following algorithm which finds the minimal dominating sets of G(Z_n, D).
If we run the algorithm, then the dominating set obtained for this graph is D = {0, 3, 11, 14, 17, 1, 2} whose cardinality is 7.
This algorithm is run by finding a minimum set of closed neighbourhood sets which cover all the vertices of G(Z_n, D).
The following algorithm finds minimal dominating sets of G(Z_n, D) for all values of n except when n is a prime. Because when n is a prime, the graph becomes an outer Hamilton cycle and hence the dominating sets are found easily.

Algorithm – DS - DCG:
INPUT : Enter a number
OUTPUT : Minimal dominating sets
STEP 1 : Enter a number n
STEP 2 : IF n IS NOT EQUAL TO NULL
GOTO STEP 3
ELSE GOTO STEP 12
STEP 3 :METHOD setofdivisiors($number){
INITIALIZE ARRAY $divisorset
FOR EACH $i,$i=1,......$i<=$number
DO THE FOLLOWING
INITIALIZE $modval
ASSIGN $number % $i TO $modval
IF $modval EQUAL TO 0
ASSIGN $i TO $divisorset[]
END IF
RETURN $divisorset
END FOR
STEP 4 : Find set of divisors
CALL METHOD setofdivisiors(param)
INITIALIZEVARIABLE$resultASSIGN
COUNTOF(setofdivisiors (param))
INITIALIZE VARIABLE $setOfDivisorsArray ASSIGN
setofdivisiors(param)
PRINT "Set of divisors ={";
FOR EACH $i,$i=0,......$i<param
INITIALIZEARRAY$setOfDivisorsArrayTemp[]ASSIGN
$setOf DivisorsArray[$i];
PRINT $setOfDivisorsArray[$i];
IF $i NOT EQUAL TO $result-1
PRINT ",";
END IF
END FOR
PRINT "}"
STEP 5 : Find Symmetric sub set
INITIALIZEVARIABLE$setOfDivisorsCount

ASSIGNCOUNTOF($setOfDivisorsArrayTemp);
INITIALIZE ARRAY $firstTempArray
INITIALIZE ARRAY $secondTempArray
FOR EACH $j,$j=0,..........$j<$setOfDivisorsCount
IF $setOfDivisorsArrayTemp[$j] NOT EQUAL TO $number
IF $setOfDivisorsArrayTemp[$j] EQUAL TO $number
BREAK
ELSE
ASSIGN VALUE $setOfDivisorsArrayTemp[$j]
TO $firstTempArray[]
ASSIGNVALUE($number$setOfDivisorsArrayTemp[$j])
TO
$second TempArray[]
END IF
END IF
END FOR
MERGE $firstTempArray AND $secondTempArray
array_merge($firstTempArray,$secondTempArray)

```
INITIALIZE VARIABLE $mergeFSTemps
REMOVEDUPLICATEVALUESUSINGarray_unique
        (array_merge ($firstTempArray,
$secondTempArray))
    ASSIGN TO  $mergeFSTemps
SORT ARRAY asort($mergeFSTemps);
INITIALIZE VARIABLE $keynum
ASSIGN 0 TO $keynum
WHILE (list($key, $val) = EACH($mergeFSTemps)) {
INITIALIZE ARRAY $symmetricSubSet
ASSIGN VALUE $val TO $symmetricSubSet[]
INCREMENT $keynum++;
END WHILE
INITIALIZE VARIABLE $symmetricSubSetCount
ASSIGN COUNT OF count($symmetricSubSet)
        TO  $symmetricSubSetCount
PRINT "S* ={"
FOR EACH $m,$m=0,..........$m<$symmetricSubSetCount
PRINT $symmetricSubSet[$m];
IF $m NOT EQUAL TO $symmetricSubSetCount-1
PRINT ",";
END IF
END FOR
PRINT "}"
```

STEP 6 : Find Neighbourhood sets of n

```
FOR EACH $nsa,$nsa=0,..........$nsa<$number
FOR EACH $nsb,$nsb=0,..........$nsb<$symmetricSubSet
INITIALIZE MULTI DIMENSIONAL ARRAY
        $neighbourhoodSetArray
ASSIGN $symmetricSubSet[$nsb]

    TO  $neighbourhoodSetArray[$nsa][$nsb]
ASSIGN $symmetricSubSet[$nsb]+1 TO
$symmetricSubSet[$nsb]
IF $symmetricSubSet[$nsb] EQUAL TO $number
ASSIGN 0 TO $symmetricSubSet[$nsb]
END IF
END FOR
END FOR
/ /FIRST ELEMENTS UNION
FOR EACH $ns,$ns=0,...........$ns<$number
PUSH TO FIRST PLACE
array_unshift($neighbourhoodSetArray
[$ns], $ns)
END FOR
PRINT "Neighbourhood sets of n are"
FOR EACH $ns,$ns=0,...........$ns<$number
PRINT "N[$ns]={"
FOR EACH $ nbs,$nbs=0,...........$nbs<count
        ($neighbourhoodSetArray[$ns]
PRINT $neighbourhoodSetArray[$ns][$nbs];
IF $nbs EQUAL TO
count($neighbourhoodSetArray[$ns])-1
PRINT ""
ELSE
PRINT ","
END FOR
PRINT "}";
END FOR
```

STEP 7 : PRINT "Consider N[0] ={";

```
FOR EACH
$m,$m=0,..........$m<count($neighbourhoodSetArray[0])
INITIALIZE ARRAY $nof0array
ASSIGN $neighbourhoodSetArray[0][$m] TO
$nof0array[]
PRINT $neighbourhoodSetArray[0][$m];
IF $m NOT EQUAL TO
count($neighbourhoodSetArray[0])-1
PRINT ",";
END IF
END FOR
PRINT "}"
```

STEP 8 : Find Uncovered vertices in N[0]

```
FOR EACH $m,$m=0,..........$m<$number
INITIALIZE ARRAY $notin
IF !in_array($m, $nof0array)
ASSIGN$m TO $notin[]
END IF
END FOR
PRINT "Uncovered vertices in N[0] are {";
FOR EACH $m,$m=0,..........$m<count($notin)
PRINT $notin[$m];
IF $m NOT EQUAL TO count($notin)-1)
PRINT ",";
END IF.
STOP
```

Theorem 2.6 : Let $n = p^m$, where $p = 2$ and $m > 2$. Then $D = \{rd_0 / 0 \leq r \leq k-1$ where k is the largest positive integer such that $rd_0 < n$ and $d_0 = 5\}$ is a dominating set of $G(Z_n, D)$.

Proof : Let $n = p^m$ where $p = 2$ and $m > 2$.

Consider the graph $G(Z_n , D)$.

In this case, $S = \{1, p, p^2 \ldots. p^m\}$ and $S^* = \{1, p, p^2 \ldots., p^m-1, p^m- p, p^m- p^2, p^m- p^3, \ldots\ldots. p^m- p^{m-1}\}$.

The graph is $|S^*|$ - regular.

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Let D = {rd_0 / 0 ≤ r ≤ k-1 where k is the largest integer such that $rd_0 < n$ and $d_0 = 5$}.
i.e., D = {0, 5, 10 ,15,……}.
We now show that D is a dominating set of G(Z_n, D).
The vertices in D are adjacent with the following vertices.
$0 \to$ 1, p, p^2 …… p^m- p, $p^m - p^2$, $p^m - p^3$, ……., $p^m - p^{m-1}$
$5 \to$ 6, p+5, $p^2 + 5$,…. $p^m + 4$, $p^m - p + 5$, $p^m - p^2 + 5$, $p^m - p^3 + 5$, …., $p^m - p^{m-1} + 5$
$10 \to$ 11, p +10, $p^2 + 10$ ……. $p^m + 9$, $p^m - p + 10$, $p^m - p + 10$, $p^m - p^3 + 10$,…. , $p^m - p^{m-1} + 10$.

If we replace p = 2 then the adjacency of the vertices is as follows.
$0 \to$ 1,2,4,8,16,…., $p^m - p^{m-1}$
$5 \to$ 6,7,9,13,21,…., $p^m - p^{m-1} + 5$
$10 \to$ 1,12,14,18,26,,…., $p^m - p^{m-1} + 10$

If we give the value for m, then the remaining set of vertices which are dominated by {0,5,10,15 ….} will be found.
Likewise, the vertices in D dominate all the vertices of the graph G(Z_n, D) and hence D is a dominating set of G(Z_n, D).

Example 2.7 : Let n = 16 i.e., p = 2 and m = 4.
Then S = {1,2,4,8,16} and S^* = {1,2,4,8,12,14,15}.
The graph of G (Z_{16}, D) is as follows.

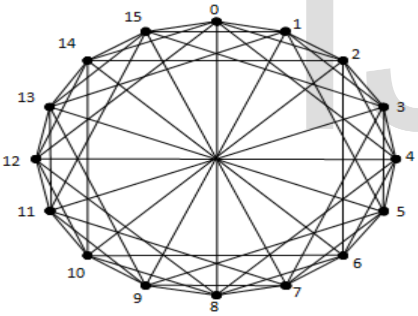

Fig. 4 : G(Z_{16}, D)

Then D becomes {0, 5, 10, 15} and the vertices adjacent with the vertices of D are
$0 \to$ 1, 2, 4, 8, 12, 14, 15
$5 \to$ 6, 7, 9, 13, 1, 3, 4
$10 \to$ 11, 12, 14, 2, 6, 8, 9
$15 \to$ 0, 1, 3, 7, 11, 13, 14
i.e., {0, 5, 10, 15} dominates all the vertices of G(Z_{16} , D). Hence this set becomes a dominating set of G(Z_{16}, D). Since G(Z_{16}, D) is 7-regular, we should have $|D| \geq 3$.
Here we have obtained $|D| = 4$. i.e., D is not minimal.

If we run the Algorithm, then the minimal dominating set obtained for this graph is D = {0, 5, 9} whose cardinality is 3.

Theorem 2.8 : If $n = p_1^{\alpha 1} p_2^{\alpha 2}$ where p_1 and p_2 are primes such that $\alpha_1, \alpha_2 \geq 1$ then D = {rd_0 / 0 ≤ r ≤ k-1 where k is the largest integer such that $rd_0 < n$ and $d_0 = 5$} is a dominating set of G(Z_n , D) .

Proof : Consider the graph G(Z_n , D) .
Let n = $p_1^{\alpha 1} p_2^{\alpha 2}$ where p_1 and p_2 are primes and $\alpha_1, \alpha_2 \geq 1$.
In this case, S = {1, p_1, p_1^2, ….., $p_1^{\alpha 1}$, p_2, p_2^2……, $p_2^{\alpha 2}$, $p_1 . p_2$, $p_1 . p_2^2$, $p_1 . p_1^3$ ……, $p_1 . p_2^{\alpha 2}$, $p_1^2 . p_2$, $p_1^3 . p_2$ …….., $p_1^{\alpha 1} . p_2$, n}.
S* = {1, p_1, p_1^2……, $p_1^{\alpha 1}$, p_2, p_2^2…….., $p_2^{\alpha 2}$, $p_1 . p_2$, $p_1 . p_2^2$, $p_1 . p_2^3$, ………. $p_1 . p_2^{\alpha 2}$, $p_1^2 . p_2$, …., $p_1^3 . p_2$, ……, $p_1^{\alpha 1} . p_2^2$, n-1, n - p, n - p_1^2 ……..,n - $p_1^{\alpha 1}$, n - p_2, n - p_2^2, ……..,
n - $p_2^{\alpha 2}$, n - $p_1 . p_2$, n - $p_1 . p_2^2$, n – $p_1 . p_2^3$,……,
n – $p_1 . p_2^{\alpha 2}$, n - $p_1^2 . p_2$, n - $p_1^3 . P_2$, ………, n - $p_1^{\alpha 1} . p_2^{\alpha 2}$ }.
The set of vertices which are adjacent with the vertices in D is expressed in the following .
$0 \to$ 1, p_1, p_1^2……, $p_1^{\alpha 1}$, p_2, p_2^2…….., $p_2^{\alpha 2}$, $p_1 . p_2$, $p_1 . p_2^2$,
$p_1 . p_2^3$, ………. $p_1 . p_2^{\alpha 2}$, $p_1^2 . p_2$, $p_1^3 . p_2$, ……, $p_1^{\alpha 1} . p_2$,
n-1, n - p, n - , p_1^2 ……..,n - $p_1^{\alpha 1}$, n - p_2, n - p_2^2, ……..,
n - $p_2^{\alpha 2}$, n - $p_1 . p_2$, n - $p_1 . p_2^2$, n – $p_1 . p_3^3$,……,
n – $p_1 . p_2^{\alpha 2}$, n - $p_1^2 . p_2$, n - $p_1^3 . P_2$, ………, n - $p_1^{\alpha 1} . p_2$
$5 \to$ 6, p_1 + 5, p_1^2 + 5, …, $p_1^{\alpha 1}$ + 5, p_2 + 5, p_2^2+5, …, $p_2^{\alpha 2}$ +5,
$p_1 . p_2$ + 5, $p_1 p_2^2$ + 5, $p_1 . p_2^3$ + 5, $p_1 . p_2^{\alpha 2}$ + 5, $p_1^2 . p_2$+5, $p_1^3 . p_2$ + 5, …, $p_1^{\alpha 1} . p_2$ + 5, n+4, n - p_1 + 5, n - p_1^2 + 5, ….,
n- $p_1^{\alpha 1}$ +5, n-p_2+5, n- p_2^2 +5, …, n- $p_2^{\alpha 2}$ +5, n- $p_1 . p_2$+5,

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n- p_1. p_2^2+5, …, n- p_1 p_1^1+5, n- p_1^2. p_2 + 5

10 → 11, p_1 + 10, p_1^2 + 10,…, $p_1^{\alpha 1}$ + 10, p_2 + 10, p_2^2+10,.. $p_2^{\alpha 2}$ +10,

p_1. p_2 + 10, p_1 . p_2^2 + 10, p_1. p_2^3 + 10, p_1. $p_2^{\alpha 2}$ + 10, p_1^2 . p_2+10, p_1^3 .p_2 + 10, …, $p_1^{\alpha 1}$. p_2 + 10, n+9, n - p_1 + 10,

n - p_1^2 + 10, …., n- $p_1^{\alpha 1}$ +10, n-p_2+10, n- p_2^2+10, …, n- $p_2^{\alpha 2}$ +10,

n- p_1. p_2+10, n- p_1. p_2^2+10, …, n- p_1 p_1^1+10, n- p_1^2. p_2 + 10.

⋮

Here we observe that the vertices {0, 5, 10,…….} are dominating the vertices 1, 6, 11, …. .

If we give the values for p_1, p_2, α_1 and α_2, then the set of vertices which are dominated by {0, 5, 10, 15….} will also be found and this set is nothing but the vertex set of G(Z_n, D).

Likewise, the vertices in D dominate the vertices of the graph G(Z_n, D) and hence D becomes a dominating set of G(Z_n, D).

Example 2.9 : For n = 12, where p_1 = 2, p_2 = 3, α_1 = 2, α_2 = 1,

S = {1,2,3,4,6,12} and S* = {1,2,3,4,6,8,9,10,11}.

The graph of G (Z_{12}, D) is as follows.

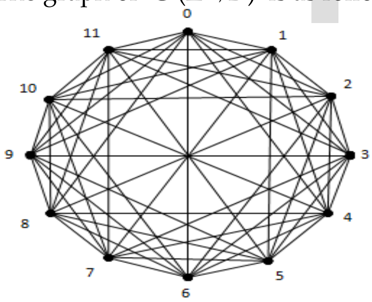

Fig.5 : G (Z_{12}, D)

Now D = {0, 5} and the vertices adjacent to D are

0 → 1, 2, 3, 4, 6, 8, 9, 10, 11

5 → 6, 7, 8, 9, 11, 1, 2, 3, 4

Thus D = {0, 5} dominates all the vertices of G(Z_{12}, D) and hence this set becomes a minimal dominating set of G(Z_{12}, D).

Example 2.10: For n = 20 where p_1=2, p_2 = 5, α_1 = 2, α_2 = 1,

S = {1,2,4,5,10, 20} and S* = {1,2,4,5,10,15,16,18,19}.

The graph of G (Z_{20}, D) is as follows.

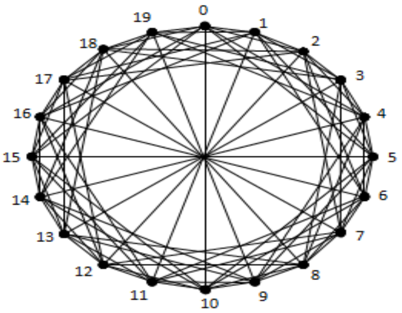

Fig.5 : G (Z_{20}, D)

Then D = {0, 5, 10, 15} and the vertices adjacent to D are

0 → 1,2,4,5,10,15,16,18,19

5 → 6,7,9,10,15,0,1,3,4

10 → 11,12,14,15,0,5,6,8,9

15 → 16,17,19,0,5,10,11,13,14

i.e., {0,5,10,15} dominates all the vertices of G(Z_{20},D).

Hence this set becomes a dominating set of G(Z_{20}, D).

Since G(Z_{20}, D) is 9-regular, we should have $|D| \geq 3$.

Here we obtained that $|D|$=4. i.e., D is not minimal.

By the Algorithm, the minimal dominating set of G(Z_{20}.D) is {0, 7, 9} whose cardinality is 3.

Theorem 2.11 : Let n = $p_1^{\alpha 1}$, $p_2^{\alpha 2}$ …….. $p_m^{\alpha m}$ where p_1, p_2 ….. p_m are primes and α_1, α_2……. $\alpha_m \geq 1$ .Then D = {rd_0 / 0 ≤ r ≤ k-1 where k is the largest positive integer such that rd_0 < n and d_0 = 6} is a dominating set of G(Z_n, D).

Proof : Let n = $p_1^{\alpha 1}$, $p_2^{\alpha 2}$ ….. $p_m^{\alpha m}$.

In this case, S = {1, p_1, .., $p_1^{\alpha 1}$, p_2, p_2^2,.., $p_2^{\alpha 2}$, ……., p_m, p_m^2 …….,$p_m^{\alpha m}$, p_1. p_2, p_1. p_2^2, …., p_1. $p_2^{\alpha 2}$, p_1^2. p_2, .., $p_1^{\alpha 1}$. $p_1^{\alpha 2}$, ……, $p_1^{\alpha 1}$ $p_2^{\alpha 2}$ …..$p_2^{\alpha m}$ }.

S*= {1, p_1, .., $p_1^{\alpha 1}$, p_2, p_2^2,.., $p_2^{\alpha 2}$,……., p_m, p_m^2 ……., $p_m^{\alpha m}$, p_1. p_2, p_1. p_2^2, …., p_1. $p_2^{\alpha 2}$, p_1^2. p_2, .., $p_1^{\alpha 1}$. $p_1^{\alpha 2}$, ……, $p_1^{\alpha 1}$ $p_2^{\alpha 2}$ …..$p_2^{\alpha m}$, n – 1, n - p_1, …., n - $p_1^{\alpha 1}$,……………………… , n- $p_{m-1}^{\alpha m-1}$. $p_m^{\alpha m}$ }.

The graph is $|S^*|$ - regular.

Let D = {rd_0 / 0 ≤ r ≤ k-1 where k is the largest positive integer such that rd_0 < n and d_0 = 6}.

We now show that D is a dominating set of G(Z_n, D).

The set of vertices adjacent to the vertices of D are given below.

$0 \rightarrow 1,\ p_1,\ ..,\ p_1^{\alpha 1},\ p_2,\ p_2^2,..,\ p_2^{\alpha 2},\ ....,\ p_m,\ p_m^2......,\ p_m^{\alpha m},\ p_1.\ p_2,\ p_1.\ p_2^2,\ ....,$

$p_1.\ p_2^{\alpha 2},\ p_1^2.\ p_2,\ ..,\ p_1^{\alpha 1}.\ p_1^{\alpha 2},\ ......,\ p_1^{\alpha 1}\ p_2^{\alpha 2}\ .....\ p_2^{\alpha m},$

$n - 1,\ n - p_1,\ ....,\ n - p_1^{\alpha 1},......................... ,\ n- p_{m-1}^{\alpha m-1}\ .\ p_m^{\alpha m}$

$6 \rightarrow 7,\ p_1 + 6,\ ....,\ p_1^{\alpha 1} + 6....,\ p_2 + 6,\ p_2^2 + 6......\ p_2^{\alpha 2} + 6,\ ....$

$.......,\ p_{m-1}^{\alpha m-1}\ .\ p_m^{\alpha m} + 6,\ n + 5,\ n- p_1 + 6,\ ....,\ n- p_1^{\alpha 1} + 6....,$

$...,\ n- p_2 + 6,\ n- p_2^2 + 6......,\ n- p_{m-1}^{\alpha m-1}\ .\ p_m + 6.$

$12 \rightarrow 13,\ p_1 + 12,\ ..,\ p_1^{\alpha 1} + 12....,\ p_2 + 12,\ ..,\ p_2^{\alpha 2} + 12,......,$

$....\ p_{m-1}^{\alpha m-1}\ .\ p_m^{\alpha m} + 12,\ n + 11,.....,\ n- p_{m-1}^{\alpha m-1}\ .\ p_m + 12.$

$\vdots$

Here we observe that the vertices {0,6,12,18,….} are dominating the vertices 0,1,5,7,…

It we give the value for $p_1, p_2, ..., p_m$ and $\alpha_1, \alpha_2, ..., \alpha_m$ then the rest of the vertices which are dominated by {0,6,12,….} will also be found.

Likewise, the vertices in D dominate the vertices of the graph $G(Z_n, D)$ and hence D is a dominating set of $G(Z_n, D)$.

Example 2.12 : Let n = 42. Here $p_1 = 2$, $p_2 = 3$, $p_3 = 7$, $\alpha_1 = \alpha_2 = \alpha_3 = 1$.

Then S = {1,2,3,6,7,14,21,42} and S* ={1,2,3,6,7,14,21,28,35,36,39,40,41}.

The graph of $G(Z_{42}, D)$ is as follows.

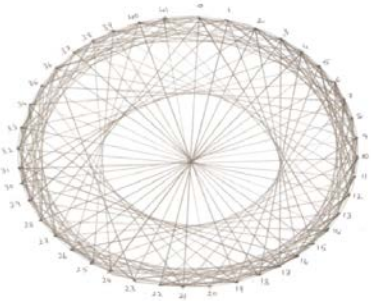

Fig. 7 : G (Z_{42}, D)

Then D becomes {0, 6, 12, 18, 24, 30,36} and the vertices adjacent with the vertices of D are

0 → 1,2,3,6,7,14,21,28,35,36,39,40,41

6 → 7,8,9,12,13,20,27,34,41,0,3,4,5

12 → 13,14,15,18,19,26,33,40,5,6,9,10,11

18 → 19,20,21,24,25,32,39,4,11,12,15,16,17

24 → 25,26,27,30,31,38,3,10,17,18,21,22,23

30 → 31,32,33,36,37,2,9,16,23,24,27,28,29

36 → 37,38,39,0,1,8,15,22,29,30,33,34,35

i.e., D = {0,6,12,18,24,30,36} dominates all the vertices of $G(Z_{42}, D)$.

Hence this set becomes a dominating set of $G(Z_{42}, D)$.

Since $(G(Z_{42}, D)$ is 13-regular, we should have $|D| \geq 4$.

Here we have obtained $|D| = 7$.

i.e., D is not minimal.

By the Algorithm, the minimal dominating set of $G(Z_{42}, D)$ is {0,10,26,16,6} whose cardinality is 5.

The given algorithm is developed by using the PHP software (server scripting language) and the following illustrations are obtained by simply giving the value for n.

2.2 Algorithm - Illustrations

1. $n = p^2$, $p \neq 2$ be a prime.

Let n =25

S ={1,5,25}

S* ={1,5,20,24}

Neighbourhood sets of $G(Z_{25}, D)$ are

N[0]={0,1,5,20,24}
N[1]={1,2,6,21,0}
N[2]={2,3,7,22,1}
N[3]={3,4,8,23,2}
N[4]={4,5,9,24,3}
N[5]={5,6,10,0,4}
N[6]={6,7,11,1,5}
N[7]={7,8,12,2,6}
N[8]={8,9,13,3,7}
N[9]={9,10,14,4,8}
N[10]={10,11,15,5,9}
N[11]={11,12,16,6,10}
N[12]={12,13,17,7,11}
N[13]={13,14,18,8,12}
N[14]={14,15,19,9,13}
N[15]={15,16,20,10,14}
N[16]={16,17,21,11,15}
N[17]={17,18,22,12,16}
N[18]={18,19,23,13,17}
N[19]={19,20,24,14,18}
N[20]={20,21,0,15,19}
N[21]={21,22,1,16,20}
N[22]={22,23,2,17,21}

N[23]={23,24,3,18,22}
N[24]={24,0,4,19,23}
Minimal dominating set is ={0,3,11,14,17,1,2}.
The number of minimal dominating sets of G(Z_{25} , D) are
{0,3,11,14,17,1,2},{1,4,12,15,18,2,3},{2,5,13,16,19,3,4},{3,6,14,17,20,4,5},
{4,7,15,18,21,5,6},{5,8,16,19,22,6,7},{6,9,17,20,23,7,8},{7,10,18,21,24,8, 9},
{8,11,19,22,0,9,10},{9,12,20,23,1,10,11},{10,13,21,24,2,11,12}, {11,14,22,0,3,12,13},{12,15,23,1,4,13,14},{13,16,24,2,5,14,15},{14,17,0,3,6,15,16},{15,18,1,4,7,16,17},{16,19,2,5,8,17,18},{17,20,3,6,9,18,19}, {18,21, 4,7,10,19,20},{19,22,5,8,11,20,21},{20,23,6,9,12,21,22},{21,24,7,10,13,22,23},{22,0,8,11,14,23,24},{23,1,9,12,15,24,0}.
{24,2,10,13,16,0,1}.
The domination number is 7.

2. n = p^m where p = 2 and m > 2.

Let n =16
S ={1,2,4,8,16}
S* ={1,2,4,8,12,14,15}
Neighbourhood sets of G(Z_{16} , D) are
N[0]={0,1,2,4,8,12,14,15}
N[1]={1,2,3,5,9,13,15,0}
N[2]={2,3,4,6,10,14,0,1}
N[3]={3,4,5,7,11,15,1,2}
N[4]={4,5,6,8,12,0,2,3}
N[5]={5,6,7,9,13,1,3,4}
N[6]={6,7,8,10,14,2,4,5}
N[7]={7,8,9,11,15,3,5,6}
N[8]={8,9,10,12,0,4,6,7}
N[9]={9,10,11,13,1,5,7,8}
N[10]={10,11,12,14,2,6,8,9}
N[11]={11,12,13,15,3,7,9,10}
N[12]={12,13,14,0,4,8,10,11}
N[13]={13,14,15,1,5,9,11,12}
N[14]={14,15,0,2,6,10,12,13}
N[15]={15,0,1,3,7,11,13,14}
Minimal dominating set is ={0,5,9}
The number of minimal dominating sets of G(Z_{16} , D) are
{0,5,9},{1,6,10},{2,7,11},{3,8,12},{4,9,13},{5,10,14},{6,11,15},{7,12,0},
{8,13,1},{9,14,2},{10,15,3},{11,0,4},{12,1,5},{13,2,6},{14,3,7},{15,4,8}.
The domination number is 3.

3. $n = p_1^{\alpha 1}\ p_2^{\alpha 2}$ where p_1 and p_2 are primes such that $\alpha_1, \alpha_2 \geq 1$.

Let n =20
S ={1,2,4,5,10,20}
S* ={1,2,4,5,10,15,16,18,19}
Neighbourhood sets of G(Z_{20} , D) are
N[0]={0,1,2,4,5,10,15,16,18,19}
N[1]={1,2,3,5,6,11,16,17,19,0}
N[2]={2,3,4,6,7,12,17,18,0,1}
N[3]={3,4,5,7,8,13,18,19,1,2}
N[4]={4,5,6,8,9,14,19,0,2,3}
N[5]={5,6,7,9,10,15,0,1,3,4}
N[6]={6,7,8,10,11,16,1,2,4,5}
N[7]={7,8,9,11,12,17,2,3,5,6}
N[8]={8,9,10,12,13,18,3,4,6,7}
N[9]={9,10,11,13,14,19,4,5,7,8}
N[10]={10,11,12,14,15,0,5,6,8,9}
N[11]={11,12,13,15,16,1,6,7,9,10}
N[12]={12,13,14,16,17,2,7,8,10,11}
N[13]={13,14,15,17,18,3,8,9,11,12}
N[14]={14,15,16,18,19,4,9,10,12,13}
N[15]={15,16,17,19,0,5,10,11,13,14}
N[16]={16,17,18,0,1,6,11,12,14,15}
N[17]={17,18,19,1,2,7,12,13,15,16}
N[18]={18,19,0,2,3,8,13,14,16,17}
N[19]={19,0,1,3,4,9,14,15,17,18}
Minimal dominating set is ={0,7,9}
The number of minimal dominating sets of G(Z_{20} , D) are
{0,7,9},{1,8,10},{2,9,11},{3,10,12},{4,11,13},{5,12,14},{6,13,15},{7,14, 16},
{8,15,17},{9,16,18},{10,17,19},{11,18,0},{12,19,1},{13,0,2},{14,1,3}, {15,2,4}, {16,3,5},{17,4,6},{18,5,7},{19,6,8}.
The domination number is 3.

4. Let n = $p_1^{\alpha 1} . p_2^{\alpha 2} \ldots\ldots p_m^{\alpha m}$ where p_1, p_2 ….. p_m are primes and $\alpha_1, \alpha_2 \ldots\ldots, \alpha_m \geq 1$

Let n =42
S ={1,2,3,6,7,14,21,42}
S* ={1,2,3,6,7,14,21,28,35,36,39,40,41}
Neighbourhood sets of G(Z_{42} , D) are
N[0]={0,1,2,3,6,7,14,21,28,35,36,39,40,41}
N[1]={1,2,3,4,7,8,15,22,29,36,37,40,41,0}
N[2]={2,3,4,5,8,9,16,23,30,37,38,41,0,1}
N[3]={3,4,5,6,9,10,17,24,31,38,39,0,1,2}
N[4]={4,5,6,7,10,11,18,25,32,39,40,1,2,3}
N[5]={5,6,7,8,11,12,19,26,33,40,41,2,3,4}
N[6]={6,7,8,9,12,13,20,27,34,41,0,3,4,5}

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N[7]={7,8,9,10,13,14,21,28,35,0,1,4,5,6}
N[8]={8,9,10,11,14,15,22,29,36,1,2,5,6,7}
N[9]={9,10,11,12,15,16,23,30,37,2,3,6,7,8}
N[10]={10,11,12,13,16,17,24,31,38,3,4,7,8,9}
N[11]={11,12,13,14,17,18,25,32,39,4,5,8,9,10}
N[12]={12,13,14,15,18,19,26,33,40,5,6,9,10,11}
N[13]={13,14,15,16,19,20,27,34,41,6,7,10,11,12}
N[14]={14,15,16,17,20,21,28,35,0,7,8,11,12,13}
N[15]={15,16,17,18,21,22,29,36,1,8,9,12,13,14}
N[16]={16,17,18,19,22,23,30,37,2,9,10,13,14,15}
N[17]={17,18,19,20,23,24,31,38,3,10,11,14,15,16}
N[18]={18,19,20,21,24,25,32,39,4,11,12,15,16,17}
N[19]={19,20,21,22,25,26,33,40,5,12,13,16,17,18}
N[20]={20,21,22,23,26,27,34,41,6,13,14,17,18,19}
N[21]={21,22,23,24,27,28,35,0,7,14,15,18,19,20}
N[22]={22,23,24,25,28,29,36,1,8,15,16,19,20,21}
N[23]={23,24,25,26,29,30,37,2,9,16,17,20,21,22}
N[24]={24,25,26,27,30,31,38,3,10,17,18,21,22,23}
N[25]={25,26,27,28,31,32,39,4,11,18,19,22,23,24}
N[26]={26,27,28,29,32,33,40,5,12,19,20,23,24,25}
N[27]={27,28,29,30,33,34,41,6,13,20,21,24,25,26}
N[28]={28,29,30,31,34,35,0,7,14,21,22,25,26,27}
N[29]={29,30,31,32,35,36,1,8,15,22,23,26,27,28}
N[30]={30,31,32,33,36,37,2,9,16,23,24,27,28,29}
N[31]={31,32,33,34,37,38,3,10,17,24,25,28,29,30}
N[32]={32,33,34,35,38,39,4,11,18,25,26,29,30,31}
N[33]={33,34,35,36,39,40,5,12,19,26,27,30,31,32}
N[34]={34,35,36,37,40,41,6,13,20,27,28,31,32,33}
N[35]={35,36,37,38,41,0,7,14,21,28,29,32,33,34}
N[36]={36,37,38,39,0,1,8,15,22,29,30,33,34,35}
N[37]={37,38,39,40,1,2,9,16,23,30,31,34,35,36}
N[38]={38,39,40,41,2,3,10,17,24,31,32,35,36,37}
N[39]={39,40,41,0,3,4,11,18,25,32,33,36,37,38}
N[40]={40,41,0,1,4,5,12,19,26,33,34,37,38,39}
N[41]={41,0,1,2,5,6,13,20,27,34,35,38,39,40}

Minimal dominating set is ={0,10,26,16,6}

The number of minimal dominating sets of G(Z_{42} , D) are {0,10,26,16,6},{1,11,27,17,7},{2,12,28,18,8},{3,13,29,19,9},{4,14,30,20, 10},{5,15,31,21,11},{6,16,32,22,12},{7,17,33,23,13},{8,18,34,24, 14},{9,19,35,25,15},{10,20,36,26,16},{11,21,37,27,17},{12,22,38,28,18},{13,23,39,29,19},{14,24,40,30,20},{15,25,41,31,21},{16,26,0,32,22},{17,27,1,33,23},{18,28,2,34,24},{19,29,3,35,25},{20,30,4,36,26},{21,31,5,37,27},{22,32,6,38,28},{23,33,7,39,29},{24,34,8,40,30},{25,35,9,41,31},{26,36,10,0,32},{27,37,11,1,33},{28,38,12,2,34},{29,39,13,3,35},{30,40,14,4,36},{31,41,15,5,37},{32,0,16,6,38},{33,1,17,7,39},{34,2,18,8,40},{35,3,19,9,41},{36,4,20,10,0},{37,5,21,11,1},{38,6,22,12,2},{39,7,23,13,3},{40,8,24,14,4},{41,9,25,15,5}.

The domination number is 5.

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